Quantum gravity effects in the CGHS model of collapse to a black hole

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ABSTRACT We show that only a sector of the classical solution space of the CGHS model describes formation of black holes through collapse of matter. This sector has either right or left moving matter. We describe the sector which has left moving matter in canonical language. In the nonperturbative quantum theory all operators are expressed in terms of the matter field operator which is represented on a Fock space. We discuss existence of large quantum fluctuations of the metric operator when the matter field is approximately classical. We end with some comments which may pertain to Hawking radiation in the context of the model.

I   Introduction

We regard the CGHS model [1] as a 2d classical field theory in its own right rather than as string inspired. We view the dilaton field as just another classical field rather than as describing the string coupling. The theory shares two important features with 4d general relativity - it is a nonlinear diffeomorphism invariant field theory whose solutions describe spacetime metrics and *some* of these solutions correspond to black hole formation through matter collapse. Since the model is classically exactly solvable and modulo certain very important qualifications, has been (non-perturbatively) canonically quantized via the Dirac procedure [2], we use it as a toy model for these features of 4d quantum general relativity.

In this work we prove some results regarding the properties of classical solutions to the model using a general relativist's point of view. Next, we discuss some quantum properties of the spacetime geometry along the lines of [3]. We end with a few speculative remarks concerning Hawking radiation in the model. The discussion and analysis of the quantum mechanics of the model is based on the recent work [2].

The outline of the paper is as follows. In order to interpret the quantum theory of [2] it is essential to understand the classical solution space [4]. In section II, we show that 'most' classical solutions *do not* correspond to matter collapsing to form a black hole. More specifically, we show that if both left and right moving matter is present, the spacetime does not represent black hole formation through matter collapse.[1] However, if only 'one sided' matter is present, it is possible to obtain solutions describing collapse to a black hole.

In fact, without the restriction to one sided collapse, it is difficult to characterise broad properties of the spacetime in terms of properties of the matter field distribution (see, however [4]). We have very little control over the solution space and do not understand exactly what facets of 4d general relativistic physics, if any, are modelled by the solutions. In order to retain the solutions corresponding to the collapsing black hole spacetimes as well as to have better control on the space of solutions, we restrict attention to the one sided collapse sector in the remainder of the paper.

Solutions to the CGHS model are most simply described in Kruskal like null cone coordinates $X^{\pm}$ [2]. The one sided collapse solutions which we analyse describe a single black hole spacetime and correspond to the restriction $X^{\pm} > 0$. These solutions can be analytically extended through the entire $X^{\pm}$ plane (indeed, the quantum theory of [2] seems to require consideration of such extensions!). We show, through Penrose diagrams, how the physical spacetime is embedded in, and analytically extended to, the full $X^{\pm}$ plane. This completes our analysis of the classical solution space.

In section III, we turn to the Hamiltonian description of the model. Since we are interested in the 1 sided collapse situation, we restrict the description in [2] suitably, by setting the left mass of the spacetime and the right moving matter modes to zero. We adapt the quantization of [2] to the 1 sided collapse case. To make contact with the semiclassical treatment of Hawking radiation in the literature (see, for example,

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[1]A similar result is asserted in section 8 of [4].

[5]), left *and* right moving modes are needed. Since one set of modes have been frozen in our analysis, we do not discuss Hawking radiation related issues except for some comments at the end of the paper in section V. Instead, we focus on other issues arising in quantum gravity. We calculate quantum fluctuations of the metric operator when the matter fields are approximately classical (metric operator fluctuations have been discussed earlier in [6] and, in the context of spacetimes with an internal boundary, in [7]). We show that large quantum gravity effects as in the case of cylindrical waves [3] are manifested even far away from the singularity (although not at spatial infinity).

The discussion of this section pertains to the quantum version of the analytic extension to the entire $X^{\pm}$ plane, of the 1 sided collapse solutions. In contrast, in section IV, we deal with (the canonical classical and quantum theory of) only the physical spacetime region $X^{\pm} > 0$. To do this we appropriately modify the analysis of asymptotics in [2]. The most direct route to the quantum theory is to first gauge fix (a la Mikovic [6]) and then quantize the resulting description. We obtain a Fock space representation based on the time choice $\ln(\frac{X^+}{X^-})$ in contrast to the Fock space

of section III which was based on the time choice $\frac{X^++X^-}{2}$. We repeat the analysis of section III regarding large quantum gravity effects. In the process we find that the operator corresponding to the spacetime metric at large values of X^+ cannot be represented on the Fock space of the quantum theory.The implication is that the most natural representation (which we have chosen) for the quantum theory may not be the correct one. We leave this as an open problem.

Section V contains concluding remarks including some comments on Hawking radiation in the context of the model.

We do not attempt to review the vast amount of pertinent literature but instead refer the reader to review articles such as [5].

Notation

Besides standard conventions, we will use the following notation (from [2]) throughout this paper: In the double null coordinates $X^{\alpha} = (X^+, X^-)$, many quantities depend only on X^+ or X^-, but not on both variables. We will emphasize this by using only X^+ or X^- as an argument of that function or functional. For example, while $f(X)$ means that f is a function of both X^+ and X^-, $f_{,+}(X^+)$ and $f_{,-}(X^-)$ mean that the derivatives $f_{,+}$ and $f_{,-}$ depend only on X^+ and X^-, respectively. Moreover, $f_{,\pm}(X^{\pm})$ will serve as a shorthand notation to denote the function dependence of both $f_{,+}$ and $f_{,-}$ simultaneously. $\mathcal{I}_L^-, \mathcal{I}_R^-, \mathcal{I}_L^+$ and $\mathcal{I}_R^+$ denote past left, past right, future left and future right null infinity respectively.

II Analysis of the classical solution space

II.1 The Action and the solution to the field equations

We briefly recall the action and the solution to the field equations for the CGHS model in the notation of [2] (for details see [2]). In units in which the velocity of light, c, and the gravitational constant, G, are unity, the action is

$$S[y, \gamma_{\alpha\beta}, f] = \frac{1}{2} \int d^2X \ |\gamma|^{\frac{1}{2}} \left(yR[\gamma] + 4\kappa^2 - \gamma^{\alpha\beta} f_{,\alpha} f_{,\beta} \right) \ . \tag{1}$$

Here y is the dilaton field, $\gamma_{\alpha\beta}$ is the spacetime metric (signature $(-+)$), and f is a conformally coupled scalar field. $R[\gamma]$ denotes the scalar curvature of $\gamma_{\alpha\beta}$, and κ is a positive definite constant having the dimensions of inverse length.

To interpret the theory, we will treat $\gamma_{\alpha\beta}$ as an auxiliary metric and

$$\bar{\gamma}_{\alpha\beta} := y^{-1} \gamma_{\alpha\beta} \tag{2}$$

as the physical "black hole" metric. Since y is a conformal factor, it is restricted to be positive. However, note that since the field equations and (1) are well defined for $y \leq 0$, solutions with positive y admit analytic extensions to $y \leq 0$. The solution to the field equations arising from (1) is as follows (for details see [2]). $\gamma_{\alpha\beta}$ is flat. The remaining fields are most elegantly described in terms of the double null coordinates $X^\pm = Z \pm T$, where (Z, T) are the Minkowskian coordinates associated with the flat auxilliary metric. Then the spacetime line element associated with the metric $\gamma_{\alpha\beta}$ is

$$ds^2 = dX^+ dX^- \ , \tag{3}$$

the matter field is the sum of left and right movers

$$f(X) = f_+(X^+) + f_-(X^-) \ , \tag{4}$$

and in the conformal gauge [1]

$$y(X) = \kappa^2 X^+ X^- + y_+(X^+) + y_-(X^-) \ . \tag{5}$$

Here

$$y_\pm(X^\pm) = - \int^{X^\pm} d\bar{X}^\pm \int^{\bar{X}^\pm} d\bar{\bar{X}}^\pm \left(f_{,\pm}(\bar{\bar{X}}^\pm) \right)^2 \ . \tag{6}$$

Finally, the line element corresponding to the physical metric, $\bar{\gamma}_{\alpha\beta}$ is

$$d\bar{s}^2 = \frac{dX^+ dX^-}{y} \ . \tag{7}$$

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Its scalar curvature is

$$\bar{R} = 4 \left(\frac{y_{,+-}}{y} - \frac{y_{,+}y_{,-}}{y^2} \right) . \tag{8}$$

For smooth matter fields, it is easy to see that curvature singularities can occur only when $y = 0$ or $y = \infty$ (the converse may not be true).

II.2 Unphysical nature of solutions with 'both sided matter'

We now analyse the physical spacetime structure corresponding to the solutions described above. We are interested in those solutions which describe matter collapse to a black hole. So for spacetimes of physical interest we require that
(i) A notion of (left past and future, right past and future) null infinities exists such that any light ray originating within the physical spacetime, traversing a region of no curvature singularities and reaching null infinity should exhaust infinite affine parameter to do so. Further, null infinity is the locus of all such points. Each of left past, right past, left future and right future null infinity is a null surface diffeomorphic to the real line and forms part of the boundary of the spacetime.
(ii) Only future singularities should exist. Note that since y is a conformal factor (7), it is required to be positive. Any region *within* the physical spacetime where $y \leq 0$ is defined to be singular.

For simplicity we restrict the spacetime topology to be R^2. We also assume that the matter fields be of compact support at past null infinity.

Since the null infinities are null boundaries of the spacetime, they are labelled by lines of constant $X^\pm$ (the constant could be finite or infinite). Thus, the physical spacetime is a subset of the entire Minkowskian plane framed by boundaries made up of lines of constant $X^\pm$ and a future singularity. With this picture in mind let us further analyse the consequences of **(i)** and **(ii)**.

Further consideration of **(i)** results in the following lemma.

Lemma: If a section of null infinity is labelled by $X^+ = 0$ or $X^- = 0$ then **(i)** implies that $y = 0$ on this section.
Proof: Let a section of (right future or left past) null infinity be labelled by $X^+ = 0$. Approach $X^+ = 0$, through a nonsingular region along an $X^- = \text{constant} = a^-$ line from $X^+ = a^+$ (a^+ is finite). Let the normal to this line be $k_a = \alpha(dX^-)_a$. For this line to be a geodesic $\frac{\partial \alpha}{\partial X^+} = 0$. Choose $\alpha = 1$. Let the affine parameter along this geodesic be λ. **(i)** implies

$$|\lambda(X^- = a^-, X^+ = 0) - \lambda(a^-, a^+)| = | \int_{a^+}^0 \frac{dX^+}{\alpha y} | = \infty. \tag{9}$$

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From **(i)**, if $X^+ = 0$ is to label null infinity, $y \to 0$ as $X^+ \to 0$ in such a way as to make the integral diverge. Hence $y(a^-, 0) = 0$.

We now show that **(i)** or **(ii)** is violated if both left and right moving matter is present. For this, we examine (5), (6) and choose the lower limits of integration in (6) as follows. Let the least value of X^- be X_0^- on left future null infinity and that of X^+ be X_0^+ on right past null infinity. Then we specify y as

$$y(X) = \kappa^2 X^+ X^- + y_+(X^+) + y_-(X^-) + a_+ X^+ + a_- X^- + b. \qquad (10)$$

where $a_\pm, b$ are constants and

$$y_\pm(X^\pm) = - \int_{X_0^\pm}^{X^\pm} d\bar{X}^\pm \int_{X_0^\pm}^{\bar{X}^\pm} d\bar{\bar{X}}^\pm \left(f_{,\pm}(\bar{\bar{X}}^\pm) \right)^2 . \qquad (11)$$

The auxilliary flat metric determines $X^\pm$ only upto Poincare transformations. In this section, if X_0^+ (X_0^-) happens to be finite, we use the translational freedom in X^+ (X^-) to set $X_0^+ = 0$ ($X_0^- = 0$).

Our strategy will be to demand **(i)** or **(ii)** and use the lemma for exhaustive choices of ranges of $X^\pm$. Thus we *assume* that the physical spacetime satifies **(i)**, **(ii)** and that the boundaries of these ranges label the infinities of the spacetime. Singularities will occur inside the ranges when $y < 0$:

(A) $-\infty < X^\pm < \infty$: Past timelike infinity is labelled by $(X^-, X^+) = (\infty, -\infty)$. As we approach this point, the first term on the right hand side of (10) becomes arbitrarily negative and since it dominates the behaviour of y, it drives y to negative values. The region $y < 0$ must 'intersect' past left and past right infinity. Since y cannot be negative, there must be a past singularity in the spacetime. Thus **(ii)** rules out this range for $X^\pm$.

(B) $-\infty < X^- < \infty$, $0 < X^+ < \infty$: Left past null infinity is labelled by $X^+ = 0$. By the Lemma, $y(X^-, 0) = 0$. From (10) this gives, on left past null infinity

$$y^-(X^-) + a_- X^- + b = 0 \qquad (12)$$

Differentiating this equation with respect X^- yields $f_{,-} = 0$. Thus **(i)** implies that there cannot be right moving matter for this choice of range.

All other choices of range can be handled by using the arguments in **(A)**,**(B)**. The conclusion is that either there is a past singularity in the spacetime so that the corresponding range is ruled out or $f_{,-}$ or $f_{,+}$ vanish. Thus we have proved the following statement:

In the conformal gauge, if (i) and (ii) hold, then either left moving or right moving matter must vanish.

Note that we have *not* proved the converse of this statement.

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II.3 One sided collapse to a black hole

Having established that classical solutions of physical interest contain only 'one sided' matter, we turn to the analysis of (10) with $f^- = 0$ (a similar analysis can be done for $f^+ = 0$).

We first identify the region of the $X^\pm$ plane corresponding to the physical space-time. Let us fix the translation freedom in $X^\pm$ by setting $a_\pm = 0$ in (10) [2]. Using arguments similar to those in the Lemma, **(A)** and **(B)**, it can be shown that the only possible labellings of past null infinity which do not contradict **(i)**, **(ii)** and $f_{,+} \neq 0$ are past left null infinity at $X^+ = 0$ and past right null infinity at $X^- = \infty$.

Thus the solution of interest for the rest of the paper is

$$y(X) = \kappa^2 X^+ X^- + y_+(X^+) . \tag{13}$$

with

$$y_+(X^+) = - \int_0^{X^+} d\bar{X}^+ \int_0^{\bar{X}^+} d\bar{\bar{X}}^+ \left(f_{,+}(\bar{\bar{X}}^+) \right)^2 , \tag{14}$$

and $X^+ > 0$. Let the support of f^+ be $\alpha < X^+ < \beta$. Note that within the physical spacetime $X^- \geq 0$ otherwise $y(x)$ can become negative. Note that

$$y(x) = \kappa^2 X^+ X^- \quad \text{for} \quad X^+ < \alpha, X^- \geq 0, \tag{15}$$

and the spacetime is flat. For this region the null line $X^- = 0$ is part of $\mathcal{I}_L^+$. Similarly, $\mathcal{I}_R^-$ is found to be $X^- = \infty$. Consideration of $X^+ > \beta$ fixes $\mathcal{I}_R^+$ to be at $X^+ = \infty$.

Next we examine the locus of the singularity:

$$y(x) = 0 \Rightarrow \quad \kappa^2 X^+ X^- = \int_0^{X^+} d\bar{X}^+ \int_0^{\bar{X}^+} d\bar{\bar{X}}^+ \left(f_{,+}(\bar{\bar{X}}^+) \right)^2 \tag{16}$$

$$\Rightarrow X^- = \quad \frac{1}{\kappa^2 X^+} \int_0^{X^+} d\bar{X}^+ \int_0^{\bar{X}^+} d\bar{\bar{X}}^+ \left(f_{,+}(\bar{\bar{X}}^+) \right)^2 . \tag{17}$$

The singularity intersects $\mathcal{I}_L^+$ at $(X^- = 0, X^+ = \alpha)$. It can be checked that the normal $n_a = \partial_a y$ to the curve corresponding to the singularity has norm in the auxillary metric given by

$$n^a n_a = \kappa^2 \left(\int_0^{X^+} d\bar{X}^+ \int_0^{\bar{X}^+} d\bar{\bar{X}}^+ \left(f_{,+}(\bar{\bar{X}}^+) \right)^2 - X^+ \int_0^{X^+} d\bar{X}^+ \left(f_{,+}(\bar{X}^+) \right)^2 \right) \tag{18}$$

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[2] This is a different choice from that used in Section 2.2

$$= -\kappa^2 \int_0^{X^+} d\bar{X}^+ \bar{X}^+ \left(f_{,+}(\bar{X}^+)\right)^2 \tag{19}$$

This is clearly negative for $X^+ > \alpha$. Thus the singularity is spacelike. From (17), the singularity 'intersects' future right null infinity at

$$X^- = \frac{1}{\kappa^2} \int_0^\infty dX^+ \left(f_{,+}(X^+)\right)^2 \; . \tag{20}$$

(20) gives the position of the horizon for the black hole formed by collapse of left moving matter.

That there is a single spacelike curve solving (17) can be seen from the following argument.[3] Consider y for fixed X^- as a function of X^+. Let $X^+ = X^+_{sing} > \alpha$ solve (17). It can be checked that for $X^+ > X^+_{sing}$, $y_{,+} < 0$. Thus, for a given X^-, $y = 0$ occurs at a single value of X^+.

This completes the discussion of the physical spacetime. As mentioned before, this solution admits an anlaytic extension to the whole $X^\pm$ plane. We now analyse this extension.

The full Minkowskian plane is divided into
(1)$X^\pm > 0$: The physical spacetime lies within this range. It has an analytic extension 'above' the singularity in which $y < 0$ and the metric acquires the signature +- instead of -+.

(2)$X^- > 0$, $X^+ < 0$: (13) gives $y = \kappa^2 X^+ X^-$. This describes a (complete) flat spacetime with $y < 0$ (there is a 'signature flip' for the analytically continued metric).

(3)$X^\pm < 0$: $y = \kappa^2 X^+ X^-$ describes a complete flat spacetime with $y > 0$.

(4)$X^- < 0$, $X^+ > 0$: Both terms on the right hand side of (13) survive and both are negative. So there is a 'signature flip' in this region with $y < 0$.

The structure in the full Minkowskian plane is shown, schematically, in figure 1.

III   Canonical description on the entire Minkowskian plane

We describe the one sided collapse situation in classical canonical language. This is acheived by switching off degrees of freedom associated with the right moving matter fields, in a consistent manner, in the description of [2]. Hence, we shall use the results and the notation, and adapt the procedures, of [2]. Rather than repeat the content of that paper here, we refer the reader to [2]. Henceforth we shall assume

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[3]I thank J. Samuel for suggesting this argument.

familiarity with that work. We shall also use results from [8] regarding the canonical transformation to the Heisenberg picture. Although that work dealt with a spacetime topology $S^1 \times R$, the transformation to the Heisenberg picture as well as other basic ideas such as the relation of canonical data with the spacetime solution of the Klein Gordon equation go through in the R^2 case which is of relevance here.

It would be straightforward, in what follows, to use the gauge fixing procedure of [6]. Unfortunately, the gauge fixing conditions (67) *in conjunction with* the asymptotic conditions of [2] result in a foliation inappropriate for the entire Minkowskian plane. More precisely, the foliation consists of boosted planes all passing through $X^+ = X^- = 0$ and does not cover the timelike wedges $X^+ X^- < 0$.[4] Such a foliation *does* cover the region $X^\pm > 0$ and this why we use it in section IV.

In what follows, x is a coordinate on the constant t spatial slice and the 1+1 Hamiltonian decomposition is in the context of a foliation of spacetime by such slices. We use notation such that for a given field $g(x,t)$, $\frac{\partial g}{\partial x}$ is denoted by g' and $\frac{\partial g}{\partial t}$ is denoted by $\dot{g}$.

III.1 Classical theory

In [2] the CGHS model is mapped to parametrized free field theory on a flat 2d spacetime. The transformation from (1) (after parametrization at infinities) is made to a description in terms of embedding variables and the (canonical form of the) action in these variables is

$$S[X^\pm, \Pi_\pm, f, \pi_f, \bar{N}, N^1; p, m_R)$$

$$= \int dt \int_{-\infty}^{\infty} dx \, \left(\Pi_+ \dot{X}^+ + \Pi_- \dot{X}^- + \pi_f \dot{f} - \bar{N}\bar{H} - N^1 H_1 \right) + \int dt \, p \dot{m}_R \ . \qquad (21)$$

Here $X^\pm$ are the embedding variables (they correspond to the light cone coordinates we have been using to describe the solution in earlier sections), $\Pi_\pm$ are their conjuagte momenta, f is the scalar field and π_f it's conjugate momentum, $\bar{N}$ and N^1 are the rescaled lapse and the shift and $\bar{H}$ and H_1 are the rescaled super-Hamiltonian and supermomentum constraints which take the form of the constraints for a parametrized massless scalar field on a 2-dimensional Minkowski spacetime. It is convenient to deal with the Virasoro combinations

$$H^\pm := \frac{1}{2}(\bar{H} \pm H_1) = \pm \Pi_\pm X^{\pm \prime} + \frac{1}{4}(\pi_f \pm f')^2 \approx 0 \ . \qquad (22)$$

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[4] This does not necessarily rule out the existence of a *different* set of asymptotic conditions, which together with the same gauge fixing conditions (67), gives a foliation which covers the entire Minkowskian plane.

m_R is the right mass of the spacetime and p, its conjugate momentum, has the interpretation of the difference between the parametrization time and the proper time at right spatial infinity with the left parametrization time chosen to agree with the left proper time. It is useful to recall from [2] that

$$y(x) = \kappa^2 X^+(x) X^-(x)$$

$$- \int_\infty^x d\bar{x}\, X^{-\prime}(\bar{x}) \int_\infty^{\bar{x}} d\bar{\bar{x}}\, \Pi_-(\bar{\bar{x}}) + \int_{-\infty}^x d\bar{x}\, X^{+\prime}(\bar{x}) \int_{-\infty}^{\bar{x}} d\bar{\bar{x}}\, \Pi_+(\bar{\bar{x}})$$

$$+ \int_{-\infty}^\infty dx\, X^+(x) \Pi_+(x) + \frac{m_R}{\kappa} \ . \tag{23}$$

Note that the right mass is related to the left mass by

$$\frac{m_L}{\kappa} = \frac{m_R}{\kappa} + \int_{-\infty}^\infty dx\, X^+(x) \Pi_+(x) - \int_{-\infty}^\infty dx\, X^-(x) \Pi_-(x) \ . \tag{24}$$

That the right mass appears in (21) rather than the left mass m_L is a matter of choice. In [2] if the authors had chosen to synchronise the right parametrization clock with the right proper time, the last term in (21) would be $\int dt \bar{p} \dot{m}_L$ where $\bar{p}$ denotes the difference between the left parametrization and the left proper time. So an action equivalent to (21) is

$$S[X^\pm, \Pi_\pm, f, \pi_f, \bar{N}, N^1; \bar{p}, m_L)$$

$$= \int dt \int_{-\infty}^\infty dx\, \left(\Pi_+ \dot{X}^+ + \Pi_- \dot{X}^- + \pi_f \dot{f} - \bar{N} \bar{H} - N^1 H_1 \right) + \int dt \bar{p} \dot{m}_L \ . \tag{25}$$

Note that $m_L, \bar{p}$ are constants of motion. To make contact with the solution in Section II.3, we first freeze the left mass to zero and simultaneously put $\bar{p} = 0$. The reduced action, without this pair,

$$S[X^\pm, \Pi_\pm, f, \pi_f, \bar{N}, N^1; \bar{p} = 0, m_L = 0)$$

$$= \int dt \int_{-\infty}^\infty dx\, \left(\Pi_+ \dot{X}^+ + \Pi_- \dot{X}^- + \pi_f \dot{f} - \bar{N} \bar{H} - N^1 H_1 \right) \tag{26}$$

reproduces the correct equations of motion. Alternatively, if one is not familiar with the parametrization at infinities procedure, it can be checked from (21) that m_R, p

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are constants of motion. We can, therefore, consistently freeze this degree of freedom by setting m_R, p equal to *constants* of motion and then simply use the reduced action (26). Since

$$\frac{m}{\kappa} := -\int_{-\infty}^{\infty} dx \; X^+(x)\Pi_+(x) + \int_{-\infty}^{\infty} dx \; X^-(x)\Pi_-(x) \tag{27}$$

commutes with the constraints, it is a constant of a motion and we can consistently set $m_R = m$ and $p = 0$.

Next, in order to have a description of the one sided collapse situation, we must set $f^- = 0$. This is done in the canonical treatment as follows. Through a Hamilton-Jacobi type of transformation we pass from the description in (26) to the Heisenberg picture [2, 8]. The new variables are the Fourier modes (they can be interpreted as determining the matter field and momentum on an initial slice) $a_\pm(k)$ ($k > 0$), their complex conjugates $a_\pm^*(k)$, the embedding variables $X^\pm(x)$ (these are unchanged) and the new embedding momenta $\bar{\Pi}_\pm$. The Fourier modes and the new embedding momenta are given by

$$a_\pm(k) = \frac{i}{2\sqrt{\pi k}} \int_{-\infty}^{\infty} (\pi_f \pm f')e^{ikX^+(x)} , \tag{28}$$

$$\bar{\Pi}_\pm = \frac{H^\pm}{X^{\pm\prime}} . \tag{29}$$

Thus, the vanishing of the constraints is equivalent to the vanishing of the new embedding momenta. The only nontrivial Poisson brackets for the new variables are

$$\{a_\pm(k), a_\pm^*(l)\} = -i\delta(k, l) \quad \{X^\pm(x), \bar{\Pi}_\pm(y)\} = \delta(x, y). \tag{30}$$

To summarize, the scalar field and momenta are replaced by their Fourier modes which can be thought of as coordinatizing their values on an initial slice given by $X^+(x) - X^-(x) = 0$. The embedding coordinates are unchanged and the new embedding momenta are essentially the old constraints.

Setting $f^- = 0$ is equivalent in the canonical language to demanding $\pi_f - f' = 0$ [8]. From (28), this is achieved by setting $a_-(k) = a_-^*(k) = 0$ and this can be done consistently, since the '+' and '-' modes are not dynamically coupled. So the final variables for the theory are $a_+(k), a_+^*(k), X^\pm(x)$ and $\bar{\Pi}_\pm(x)$. The latter vanish on the constraint surface. The connection to the variables $X^\pm, \Pi_\pm, f, \pi_f$ [2] is through (28) and (29). The $X^\pm, \Pi_\pm$ variables are themselves related to the geometric variables of interest (the dilaton and its canonically conjugate momentum, the induced metric on the spatial slice and its conjugate momentum) in [2].

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Since we are dealing with $-\infty < X^{\pm} < \infty$ [2], this analysis (and the next two sections on quantum theory) pertains to the analytically extended one sided collapse solution.

In this paper we shall examine only the dilaton field (23). As mentioned in [2], solving the constraints, $H^{\pm}$, expresses $\Pi_{\pm}$ in terms of $X^{\pm}$ and the scalar field and its momentum. Substituting this in (23) and using $\pi_f + f' = 2X^{+\prime}f_{,+}$ from [8], one is led back to the spacetime solution

$$y(X) = \kappa^2 X^+ X^- - \int_{-\infty}^{X^+} d\bar{X}^+ \int_{-\infty}^{\bar{X}^+} d\bar{\bar{X}}^+ \left(f_{,+}(\bar{\bar{X}}^+) \right)^2 \tag{31}$$

For the next section it is useful to examine the large X^+ behaviour of (31). For X^+ large enough that it is outside the support of the matter

$$y = \kappa^2 X^+ X^- - X^+ H + \frac{m_R}{\kappa} \tag{32}$$

with

$$H = \int_{-\infty}^{\infty} d\bar{X}^+ \left(f_{,+}(\bar{X}^+) \right)^2 \tag{33}$$

and m_R given by the right hand side of (27).

III.2   Quantum theory

The passage to quantum theory is straightforward. From [2], the operators $\hat{X}^{\pm}$ are represented by multiplication, $\hat{\Pi}_{\pm} = -i\hbar \frac{\delta}{\delta X^{\pm}}$ and $\hat{a}_+(k), \hat{a}_+^{\dagger}(k)$ by representation on

a Fock space. Note that $\sqrt{-k}\hat{\mathbf{a}}(-k)$, $(k > 0)$, in [2] corresponds, here, to $\hat{a}_+(k)$ and that the commutator

$$[\hat{a}_+(k), \hat{a}_+^{\dagger}(l)] = \hbar \delta(k, l) . \tag{34}$$

The imposition of the quantum version of the classical Heisenberg picture constraints leads us to the quantum Heisenberg picture, wherein states lie in the standard embedding independent Fock space. Note that the Fock space here is spanned by the restriction of the Fock basis of [2] to negative momenta because we have frozen the right moving modes.

In the next section, we show existence of large quantum gravity effects at large X^+. This involves calculation of fluctuations of operators, $\hat{Q}$, of the form

$$\hat{Q} = \int_{-\infty}^{\infty} dX^+ Q(X^+) : \left(\hat{f}_{,+}(X^+) \right)^2 : \tag{35}$$

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where $Q(X^+)$ is a c-number function, '::' refers to normal ordering and

$$\hat{f}(X^+) = \frac{1}{2\sqrt{\pi}} \int_0^\infty \frac{dk}{\sqrt{k}} \left(\hat{a}_+(k)e^{-ikX^+} + \hat{a}_+^\dagger(k)e^{ikX^+} \right) . \tag{36}$$

Consider the coherent state

$$|\psi_c> = e^{-\frac{1}{2\hbar}\int_0^\infty |c_+(k)|^2 dk} \exp\left(\int_0^\infty \frac{dk}{\hbar} c_+(k)\hat{a}_+^\dagger(k) \right) |0> \tag{37}$$

where $|0>$ is the Fock vacuum and $c_+(k)$ are the (c-number) modes of the classical field $f_c(X^+)$,

$$f_c(X^+) = \frac{1}{2\sqrt{\pi}} \int_0^\infty \frac{dk}{\sqrt{k}} \left(c_+(k)e^{-ikX^+} + c_+^*(k)e^{ikX^+} \right) . \tag{38}$$

The mean value $\bar{Q}$ of the operator $\hat{Q}$ in this coherent state is given by

$$<\psi_c|\hat{Q}|\psi_c> = \int_{-\infty}^\infty dX^+ Q(X^+) \left(f_{c,+}(X^+) \right)^2 \tag{39}$$

as expected.

The (square) of the fluctuation in $\hat{Q}$ is given by

$$(\Delta Q)^2 = <\psi_c|\hat{Q}^2|\psi_c> - \bar{Q}^2$$

$$= \frac{\hbar^2}{8\pi} \int_0^\infty dk k^3 |Q(k)|^2 + \frac{\hbar}{4} \int_0^\infty dk k |Q_f(k)|^2 \tag{40}$$

where $Q(k)$ is the Fourier transform of $Q(X^+)$ and $Q_f(k)$ is the Fourier transform of the function $Q_f(X^+) := Q(X^+)f_{c,+}(X^+)$. The Fourier transform of a function $g(X^+)$ is

$$g(k) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^\infty dX^+ e^{ikX^+} g(X^+) . \tag{41}$$

Note that by virtue of its being independent of $f_c(X^+)$ the $\hbar^2$ term in (40) is the vacuum fluctuation of $\hat{Q}$.

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III.3 Large Quantum Gravity effects

We examine the fluctuations of the dilaton field, y, which plays the role of a conformal factor for the physical metric and hence encodes all the nontrivial metrical behaviour. The expression for y simplifies at large X^+ and we shall calculate the fluctuations of $\hat{y}$ in this limit. $y(X)$ is turned into the operator $\hat{y}(X)$ by substituting the appropriate embedding dependent Heisenberg field operators (36) in (31). Similarly H and m_R are turned into the operators $\hat{H}$ and $\hat{m}_R$. Note that $y(x)$ is not a Dirac observable. However it can be turned into one using the 'evolving constants of motion' interpretation (see [9] and references therein).

Straightforward calculations result in the following expression for the ratio of the fluctuation in $\hat{y}$ to its mean value, at large X^+

$$\left(\frac{\Delta y}{\bar{y}}\right)^2 = \frac{(\Delta H)^2 + \left(\frac{\Delta m_R}{\kappa X^+}\right)^2 - \frac{1}{X^+\kappa}\left([\hat{H},\bar{\hat{m}}_R]_+ - 2\bar{H}\bar{m}_R\right)}{(\kappa^2 X^-)^2 \left(1 - \frac{\bar{H}}{\kappa^2 X^-} + \frac{\bar{m}_R}{\kappa^3 X^+ X^-}\right)^2}. \tag{42}$$

Here

$$[\hat{H},\bar{\hat{m}}_R]_+ = <\psi_c|\hat{H}\hat{m}_R + \hat{m}_R\hat{H}|\psi_c> . \tag{43}$$

To make contact with the classical solution of section II, we choose the coherent state to be such that $f_c(X^+)$ is of compact support. Further

$$f_c(X^+) = 0 \text{ for } X^+ \leq 0. \tag{44}$$

$\hat{H}$ corresponds to choosing $Q(X^+) =: H(X^+) = 1$ in (35) and $\frac{\hat{m}_R}{\kappa}$ to

$Q(X^+) =: \frac{m_R(X^+)}{\kappa} = X^+$. From (39), and the fact that $f_c(X^+)$ is of compact support, it is easy to see that $\bar{H}, \bar{m}_R$ are finite.

Using (40) it can be seen (in obvious notation) that

$$(\Delta H)^2 = \frac{\hbar}{4}\int_0^\infty dk k|H_f(k)|^2 = \hbar\int_0^\infty dk k^2|c_+(k)|^2. \tag{45}$$

Since f_c is of compact support, its Fourier modes decrease rapidly at infinity and have sufficiently good infrared behaviour that the integral above is both ultraviolet as well as infrared finite. This shows that the fluctuation in $\hat{H}$ is finite.

The fluctuation in $\hat{m}_R$ is

$$(\Delta m_R)^2 = \frac{\hbar^2}{8\pi}\int_0^\infty dk k^3|m_R(k)|^2 + \frac{\hbar}{4}\int_0^\infty dk k|m_{Rf}(k)|^2. \tag{46}$$

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Again, the fact that f_c is of compact support renders the second term on the right hand side of (46) UV and IR finite. We now argue that the first term corresponding to the vacuum fluctuation,

$$(\Delta_0 m_R)^2 := \frac{\hbar^2}{8\pi} \int_0^\infty dk k^3 |m_R(k)|^2 \tag{47}$$

is finite. Since $m_R(X^+) = \kappa X^+$, its Fourier transform $m_R(k)$ is ill defined. We calculate, instead, the vacuum fluctuation of the regulated operator $\hat{m}_R^{(D)}$ defined by setting

$$m_R^{(D)}(X^+) = \kappa X^+ e^{\frac{-(X^+)^2}{D^2}}. \tag{48}$$

We shall take the $D \to \infty$ limit at the end of the calculation to obtain the vacuum fluctuation of $\hat{m}_R = \hat{m}_R^{(\infty)}$.

Now $m_R^{(D)}(X^+)$ is a function of sufficiently rapid decrease at infinity that $(\Delta_0 m_R^{(D)})^2$ exists. It is evaluated to be

$$(\Delta_0 m_R^{(D)})^2 = \frac{\hbar^2 \kappa^2}{6} , \tag{49}$$

which is finite and independent of D. Thus the $D \to \infty$ limit can be taken and we have

$$(\Delta_0 m_R)^2 = \frac{\hbar^2 \kappa^2}{6} . \tag{50}$$

Finally, a straightforward calculation shows (43) also to be finite.
We evaluate (42) for 2 cases:
Case I Near right spatial infinity:
    Here $X^\pm \to \infty$. So

$$\left(\frac{\Delta y}{\bar{y}}\right)^2 \to 0 \tag{51}$$

as $O(\frac{1}{(X^-)^2})$. Thus, unlike the cylindrical wave case [3], there are no large quantum fluctuations of the metric near spatial infinity. This is because the leading order

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behaviour of the metric is dictated by $\kappa^2 X^+ X^-$ which is a state independent c-number function, unlike in the cylindrical wave case.

Case II $X^- = \frac{\bar{H}}{\kappa^2} - \frac{\bar{m}_R}{\kappa^3 X^+} + d$ and X^+ large:

Here d is a real parameter. It can be checked that d measures the distance in X^- from the singularity which occurs at $d = 0$ (see (17)). It is easy to see that

$$\left(\frac{\Delta y}{\bar{y}}\right)^2 = \frac{(\Delta H)^2}{\kappa^4 d^2} + O(\frac{1}{(X^+)^2}) \tag{52}$$

This expression makes *no* assumptions on the size of d. Using (45) in (52) and reinstating explicitly the factors of G [5] (and keeping $c = 1$), we find that upto leading order in X^+

$$\left(\frac{\Delta y}{\bar{y}}\right)^2 = \frac{\hbar G^2}{\kappa^4 d^2} \int_0^\infty dl l^2 |c_+(l)|^2 \tag{53}$$

$$= \frac{\hbar G}{\kappa^2 d^2} \int_0^\infty d\frac{l}{\kappa}(\frac{l}{\kappa})^2 (G\kappa |c_+(l)|^2). \tag{54}$$

Note that in $c = 1$ units, $[G] = M^{-1}L^{-1}$, $[\kappa] = L^{-1}$, $[c_+(l)] = M^{\frac{1}{2}}L$ and $[\hbar] = ML$. Thus $\hbar G$ is the dimensionless 'Planck number', and κd, $\frac{l}{\kappa}$ and $G\kappa |c_+(l)|^2$ are all dimensionless. From (54), there are large fluctuations in $\hat{y}$ when

$$\hbar G \gg \frac{\kappa^2 d^2}{\int_0^\infty d\frac{l}{\kappa}(\frac{l}{\kappa})^2 (G\kappa |c_+(l)|^2)} \tag{55}$$

This does not require d to be small! Large fluctuations can occur even if $\kappa d \gg 1$, provided the integral in (55) is large enough. Two cases when this is possible is if there are large enough number of low frequency scalar field excitations or if there is a high frequency 'blip' in the scalar field. This is very similar to what happens in [3]. Note that on a classical solution with mass m_R, the classical scalar curvature at a 'distance' d from the singularity as a function of X^+, is at large enough X^+,

$$R = \frac{m_R}{\kappa X^+ d} \tag{56}$$

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[5] Units are discussed in [2]

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and vanishes at $X^+ = \infty$.

The horizon is located (approximately) at $X_H^- := \frac{\bar{H}}{\kappa^2}$ (20). Therefore, if $d < 0$, the region under consideration lies within X_H^-; if $d > 0$ and X^+ is large enough, the region lies outside X_H^-. Thus, for states satisfying (55), large quantum fluctuations in the metric occur both within and outside the 'mean' location of the horizon. But from (20) this location itself fluctuates by $\frac{\Delta H}{\kappa^2}$! Thus, if the Planck number is much less than one, the above calculation does not show existence of large quantum fluctuations *outside* the *fluctuating* horizon. [6]

IV Canonical description on the $X^\pm > 0$ sector of the Minkowskian plane

Section III is applicable to the analytic extension of the one sided collapse situation to the full Minkowskian plane. In this section we attempt to deal *only* with the physical spacetime and not with its analytic extension. We modify the analysis of [2], pertinent to the entire Minkowskian plane $-\infty < X^\pm < \infty$ in order to treat the case when $X^\pm > 0$. This involves a modification of the asymptotics at left spatial infinity. Now, x is restricted to be positive and $x = 0$ labels left spatial infinity. As mentioned in section III, the simplest route to quantum theory is through gauge fixing [6] the description in terms of the original geometric variables rather than by transforming to embedding variables.

IV.1 Classical theory

As in the previous section we assume familiarity with [2]. The canonical form of the action in the original geometric variables is

$$S[y, \pi_y, \sigma, p_\sigma, f, \pi_f, \bar{N}, N^1]$$

$$= \int dt \int_{-\infty}^{\infty} dx \left(\pi_y \dot{y} + p_\sigma \dot{\sigma} + \pi_f \dot{f} - \bar{N}\bar{H} - N^1 H_1 \right) \tag{57}$$

with

$$\bar{H} = -\pi_y \sigma p_\sigma + y'' - \sigma^{-1}\sigma' y' - 2\kappa^2 \sigma^2 + \frac{1}{2}(\pi_f^2 + f'^2) \tag{58}$$

and

$$H_1 = \pi_y y' - \sigma p_\sigma' + \pi_f f' . \tag{59}$$

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[6]I thank Sukanta Bose for comments regarding this point.

Here π_y is the momentum conjugate to the dilaton, σ is the spatial metric (induced from the auxilliary spacetime metric) and p_σ is its conjugate momentum.

The asymptotic conditions at right spatial infinity (which corresponds to $x = \infty$) are unchanged from [2]. Left spatial infinity is labelled by $x = 0$. We require, as $x \to 0$

$$y = \kappa^2 x^2 + O(x^3) \quad \sigma = 1 + O(x^2) \tag{60}$$

$$\pi_y = O(x) \quad p_\sigma = O(x^2) \tag{61}$$

$$\bar{N} = \alpha_L x + O(x^3) \quad N^1 = O(x^3) , \tag{62}$$

where α_L is a real parameter.

(57) is augmented with surface terms to render it functionally differentiable. The result is

$$S[y, \pi_y, \sigma, p_\sigma, f, \pi_f, \bar{N}, N^1]$$

$$= \int dt \int_0^\infty dx \, \left(\pi_y \dot{y} + p_\sigma \dot{\sigma} + \pi_f \dot{f} - \bar{N}\bar{H} - N^1 H_1 \right)$$

$$+ \int dt \, \left(-\alpha_R \frac{m_R}{\kappa} \right) . \tag{63}$$

Here α_R is related to the asymptotic behaviour of the lapse at right spatial infinity[2]. It can be checked that with f, π_f of compact support, all the asymptotic conditions are preserved under evolution. Note that (60) automatically ensures that $m_L = 0$.

To make contact with the 1 sided collapse solution the right moving modes must be set to zero. We do this as follows. Note that upto total time derivatives

$$2 \int_0^\infty \pi_f \dot{f} = \int_0^\infty dx \, \left(\int_0^x \pi_-(\bar{x}) d\bar{x} \right) \dot{\pi}_-(x) - \int_0^\infty dx \, \left(\int_0^x \pi_+(\bar{x}) d\bar{x} \right) \dot{\pi}_+(x) \tag{64}$$

where

$$\pi_\pm := \pi_f \pm f' . \tag{65}$$

Thus, we can replace f, π_f by $\pi_\pm$, with the new Poisson brackets being

$$\{\pi_\pm(x), \pi_\pm(y)\} = \pm 2 \frac{d\delta(x, y)}{dx} \quad \{\pi_+(x), \pi_-(y)\} = 0 . \tag{66}$$

Since π_+ and π_- do not couple dynamically, we can consistently freeze $\pi_- = 0$. This corresponds to setting $f^- = 0$.

Now, with a view towards quantization we introduce the gauge fixing conditions [6]

$$\pi_y = 0 \quad \sigma = 1 \ . \tag{67}$$

Using (67) in the constraints, the general solution for y and p_σ, in terms of π_+, consistent with the asymptotic conditions at left and right spatial infinity is:

$$y \;=\; \kappa^2 x^2 - \int_0^x d\bar{x} \int_0^{\bar{x}} d\bar{\bar{x}} \frac{\pi_+^2(\bar{\bar{x}})}{4} \tag{68}$$

$$p_\sigma \;=\; \int_0^x d\bar{x} \frac{\pi_+^2(\bar{x})}{4} \ . \tag{69}$$

Requiring preservation of (67) under evolution along with consistency with the asymptotic conditions fixes

$$\bar{N} = \alpha x \quad N^1 = 0 \tag{70}$$

where α is a real parameter. From [2] and (68)

$$\frac{m_R}{\kappa} = \int_0^\infty dx\, x \frac{\pi_+^2(x)}{4} \ . \tag{71}$$

Substituting this in (63) and using (64), we get

$$S[\pi_+(x)] = - \int dt \left(\int_0^\infty dx \left(\int_0^x \frac{\pi_+(\bar{x})}{2} d\bar{x} \right) \dot{\pi}_+(x) - \int_0^\infty dx\, x \frac{\pi_+^2(x)}{4} \right) \ . \tag{72}$$

In the above equation put

$$\kappa r := \ln(\kappa x) \quad \bar{\pi}_+ = e^{\kappa r}\pi_+ \tag{73}$$

to get

$$S[\bar{\pi}_+(x)] = - \int dt \left(\int_{-\infty}^\infty dr \left(\int_{-\infty}^r \frac{\bar{\pi}_+(\bar{r})}{2} d\bar{r} \right) \dot{\bar{\pi}}_+(r) - \int_{-\infty}^\infty dr \frac{\bar{\pi}_+^2(r)}{4} \right) \ . \tag{74}$$

Note that the last term $(= \frac{m_R}{\kappa})$ simplifies. The equations of motion are

$$\dot{\bar{\pi}}_+(r,t) = \{\bar{\pi}_+(r,t), \frac{m_R}{\kappa}\} = \frac{\partial \bar{\pi}_+(r,t)}{\partial r} \tag{75}$$

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The appropriate mode expansion which solves this is

$$\bar{\pi}_+(r,t) = \frac{1}{\sqrt{\pi}} \int_0^\infty dk\sqrt{k} \left(-i\bar{a}_+(k)e^{-ikr^+} + i\bar{a}_+^*(k)e^{ikr^+} \right) \tag{76}$$

where $r^+ := r + t$. From (66) and (76), the only nontrivial Poisson brackets between the mode coefficients are

$$\{\bar{a}_+(k), \bar{a}_+^*(l)\} = (-i)\delta(k,l) \tag{77}$$

From [2], one can understand the slicing of the spacetime corresponding to the gauge fixing conditions (67) we have used. In particular, on a solution, one can see that (r,t) is related to $X^\pm$ by

$$\kappa X^\pm = e^{\kappa(r\pm t)} \tag{78}$$

and that the physical metric in these coordinates is manifestly asymptotically flat at the spatial infinities.

The large X^+ behaviour of y is again given by (32). Now y, H and $\frac{m_R}{\kappa}$ evaluated in the new coordinates take the form

$$y = \kappa^2 e^{2\kappa r} - e^{\kappa r^+} H + \frac{m_R}{\kappa} \ , \tag{79}$$

$$H = \int_0^\infty dX^+ \frac{(\pi_+(X^+))^2}{4} = \int_{-\infty}^\infty dr^+ e^{-\kappa r^+} \frac{(\bar{\pi}_+(r^+))^2}{4} \tag{80}$$

and

$$\frac{m_R}{\kappa} == \int_0^\infty dX^+ X^+ (\pi_+(X^+))^2 = \int_{-\infty}^\infty dr^+ \frac{\bar{\pi}_+^2(r^+)}{4} \ . \tag{81}$$

In the $X^\pm$ coordinates H (33) took the form of the conventional Hamiltonian for free field theory on the entire $X^\pm$ plane, but $\frac{m_R}{\kappa}$ was more complicated. In the (r,t) coordinates, $\frac{m_R}{\kappa}$ takes the form of the conventional Hamiltonian for free field theory on the entire (r,t) plane (this is just the $X^\pm > 0$ part of the entire $X^\pm$ plane), but H is complicated.

IV.2   Quantum theory

The mode operators $\hat{a}_+(k), \hat{a}_+^\dagger(k)$ are represented in the standard way on the Fock space with vacuum $|\bar{0}>$. They have the standard commutation relations

$$[\hat{a}_+(k), \hat{a}_+^\dagger(l)] = \hbar \delta(k, l) \ . \tag{82}$$

Following the pattern of section III.2 and III.3 we attempt to calculate the fluctuations of $\hat{y}$ in the large X^+ region, in the coherent state

$$|\psi_c> = e^{-\frac{1}{2\hbar} \int_0^\infty |\bar{c}_+(k)|^2 dk} \exp\left(\int_0^\infty \frac{dk}{\hbar} \bar{c}_+(k) \hat{a}_+^\dagger(k) \right) |\bar{0}> \tag{83}$$

corresponding to the classical field

$$\bar{\pi}_{+c}(r^+) = \frac{1}{\sqrt{\pi}} \int_0^\infty dk \sqrt{k} \left(-i\bar{c}_+(k) e^{-ikr^+} + i\bar{c}_+^*(k) e^{ikr^+} \right) \ . \tag{84}$$

which is of compact support in r^+.

Formally, (42) again expresses the fluctuations in $\hat{y}$. However, now the crucial operator is $\hat{H}$. It is obtained from the corresponding classical expression (80) in the obvious way. Similar calculations to those in section 3.2 give

$$(\Delta H)^2 \;\; = \;\; \frac{\hbar^2}{8\pi} \int_0^\infty dk k^3 |H(k)|^2 + \frac{\hbar}{4} \int_0^\infty dk k |H_{\bar{\pi}}(k)|^2 \tag{85}$$

where $H(k)$ is the Fourier transform of

$$H(r^+) := e^{-\kappa r^+} \tag{86}$$

and $H_{\bar{\pi}}(k)$ is the Fourier transform of the function

$$H_{\bar{\pi}}(r^+) := H(r^+) \frac{\bar{\pi}_{+c}(r^+)}{4} \ . \tag{87}$$

The Fourier transform of a function $g(r^+)$ is

$$g(k) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} dr^+ e^{ikr^+} g(r^+) \ . \tag{88}$$

Since $e^{-\kappa r^+}$ is not a function of rapid decrease in the r^+ variable, the first integral in (85) is ill defined. It can be regulated by introducing the regulator $e^{-\frac{(r^+)^2}{D^2}}$. The relevant regulated integral diverges in the limit $D \to \infty$ as $\frac{e^{2D^2}}{D^2}$. Thus the vacuum fluctuations of $\hat{H}$ diverge. Therefore, in this representation we cannot proceed further with the analysis and $\hat{y}$, as it stands, cannot be given meaning as an operator on the Fock space.

V Discussion

One way of describing the spacetime geometries which arise in the CGHS model is as follows. Consider the Minkowskian plane with the flat auxilliary metric (3), on which a scalar field propagates in accordance with the flat space wave equation. The spacetime metric is conformal to the auxilliary flat one. The conformal factor y is determined by the matter distribution through (5) and is required to be positive. The field equations continue to make sense for $y \leq 0$. If one removes the restriction of positivity of y, then the following picture emerges. The Minkowskian plane is divided into spacetimes each of which have $y > 0$ or $y < 0$. The former have the signature $-+$ and the latter $+-$. As far as we know, typically, $y = 0$ labels singularities or boundaries at infinity for these spacetimes and some of these singularities may be past singularities.

This is the classical picture which corresponds to the quantum theory in [2]. Among all these classical solutions there are solutions which describe black holes formed from matter collapse. It may be that the entire solution space and the associated quantum theory [2, 6, 10] is required, in order to understand issues arising in black hole formation. In particular it may be that an understanding of Hawking radiation from a nonperturbative quantum theoretic viewpoint requires a treatment as in [6].

However, in this paper we have adopted the viewpoint that only the solutions which describe the physically interesting situation of black hole formation through matter collapse are to be taken as the basis for passage to quantum theory. We have shown that these solutions have only left or right moving matter. We concentrated on the solution with left moving matter which described a collapsing black hole spacetime

in the $X^\pm > 0$ part of the plane. This solution admitted analytic extension to the full Minkowskian plane and we showed existence of large quantum gravity effects away from the singularity in a quantum theory based on this set of analytically extended solutions. Large quantum fluctuations of the metric occur even when the 'classical' curvature is small. However, since the position of the horizon also fluctuates, our calculation does not prove existence of large fluctuations outside the (fluctuating) horizon. Next, we dealt with the classical and quantum theory based on only the $X^\pm > 0$ region. Note that even within this region there is an analytic extension of the solution 'above' the singularity. In the quantum theory a quantity of interest, H (80), could not be represented as an operator on the Fock space of the theory. This is unfortunate because the classical theory captures physically relevant collapse situations (modulo the extension through the singularity). Note that $\frac{m_R}{\kappa}$ takes the form of the usual Hamiltonian for free field theory and is also a quantity of interest. We do not know if a representation of quantum scalar field theory exists such that both H and m_R can be promoted to operators.

For the quantum theory based on the entire $X^\pm$ plane there were no such difficulties. In a sense, the quantum theories on the entire plane and the $X^\pm > 0$ region are unitarily inequivalent. The former uses a positive-negative frequency split based on the time choice $T = \frac{X^+ + X^-}{2}$ and the latter on a time choice $t = \frac{1}{\kappa} \ln T$. This is very reminiscent of what happens in the Unruh effect in 2d [11], except that, there, both sets of modes are present. The role of the acceleration in the Unruh effect is taken, here, by κ.

The following comments regarding Hawking radiation are speculative.

It seems significant that the Hawking temperature to leading order in the mass, from semiclassical calculations [5] is independent of the mass and is precisely the Unruh temperature for observers accelerating with κ. This line of thought has been pursued in [12] in the semiclassical context.

Calculations of the Hawking effect seem to require both right and left moving matter. Therefore, let us switch the right moving modes on and go back to the quantization of [2]. The quantum theory is a standard unitary quantum field theory on a Fock space. But, as emphasized before, it corresponds to the analytic extension of the usual CGHS model. We beleive that it is the analytic extension which plays a key role in obtaining a unitary theory. A possibility is that the correlations in the quantum field which appear to have been lost by passage into the singularity reappear in the analytic extension beyond the singularity in the new 'universe' which lies on the 'other side' of the singularity.

Note that instead of freezing the degrees of freedom corresponding to the right moving modes, as is done in this work, one can continue to use the results of [2], but evaluate quantities pertaining to one sided collapse by restricting the right moving part of the quantum states to the (right moving) Fock vacuum. Then vacuum fluctuations of the right moving modes would contribute to various quantities but we believe that the large quantum gravity effects away from the singularity (see section II.3) will persist. Maybe, one can also examine Hawking effect issues since the right moving modes are not switched off.

Finally, from the point of view of 4d quantum general relativity, we feel that the

CGHS model could be improved to a more realistic model of black holes if somehow an internal reflecting boundary in the spacetime existed[13, 7]. The lack of such a boundary and the fact that the matter is conformally coupled so that it does not 'see' the singularities, are, we believe, the key unphysical features present in the model but absent in (the effectively 2d) spherical collapse of a scalar field in 4d general relativity. The latter is, of course a physically realistic situation, but unfortunately technically very complicated. It would be interesting to try to apply the techniques of [2] to the model with a boundary described in [13, 7] and to try to compute the metric quantum fluctuations and compare results with those in [7]. Since the boundary in [7] is itself dynamically determined, it is not clear to us to what extent the model is solvable. It would be good to have a technically solvable model which was closer to 4d collapse situations.

References

[1] C.G. Callan, S.B. Giddings, J.A. Harvey, and A. Strominger, *Phys. Rev.* **D45**, R1005 (1992).

[2] K. Kuchař, J. Romano and M. Varadarajan, *Phys.Rev.* **D55**, 795-808 (1997).

[3] A. Ashtekar, *Phys. Rev. Lett.* **77**, 4864 (1996).

[4] S. A. Hayward, *Class. Quant. Grav.* **10**, 985 (1993).

[5] A. Strominger, "Les Houches Lectures on Black Holes," e-Print Archive: hep-th/9501071 (1995).

[6] A. Miković, *Phys. Lett.* **B355**, 85 (1995).

[7] S. Das and S. Mukherji, *Phys.Rev.* **D50**, 930 (1994).

[8] K.V. Kuchař, *Phys. Rev.* **D39**, 1579-1593 (1989)

[9] S. Carlip, *Phys. Rev.* **D42**, 2647 (1990); C. Rovelli, *Phys. Rev.* **D43**, 442 (1991).

[10] D. Cangemi, R. Jackiw, and B. Zwiebach, *Ann. Phys. (NY)* **245**, 408 (1996).

[11] S. Fulling, *Phys.Rev.* **D7**, 2850 (1973).

[12] M. Cadoni and S. Mignemi, *Phys. Lett.* **B358**, 217 (1995).

[13] K. Schoutens, H. Verlinde and E. Verlinde, *Phys.Rev.* **D48**, 2670 (1993).

Figure 1: The black hole spacetime is embedded in it's analytic continuation to the entire Minkowskian plane. The curly line denotes the singularity in the black hole spacetime and the shaded region, the (left moving) matter.